Introduction to cellular automata

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It is proved that there is an automaton that can recognize the “parts”.

It is proved that the class $C_0$ is not automata recognizable.
Trap
New approach

- Is there is a way to find if a labyrinth is one or more connected?
- Try with CA??
**Cellular automation (CA)** is a 5-tuple \((X, V, O, \varphi, B)\), where:

- **Discrete cellular state space** \(X\) is a set of locally interconnected finite state automata (FSAs).
- **Local value space** \(V\) defines all possible states for each FSA. A state of each FSA can be in one of the finite number of states. The composition of all possible states of all cells create a state space of the CA.
- **Neighborhood** \(O\) is created by a set of \(k > 0\) topologically neighboring cells that can influence on a change of updated cells state, in next transition step.
- **Transition rule** \(\varphi : V \times V \times \cdots \times V \mapsto V\) determines the change of each updated cell from its current state to a new state, operating over the neighborhood.
- **Boundary condition** \(B\) can be periodic, fixed or reflecting.
Cellular automata... A *neighborhood* $N$ is created by a set of $N$ topologically neighboring cells, which are influencing a change of the state of each updated cell in the next simulation step. Typically, homogeneous neighborhoods (composed of nearest neighbors) are used.
Cellular automata...

- In general, the most difficult task in the design of CAs is to find a transition rule that will describe the temporal evolution of a modeled system (i.e. which leads to desired global response of the system).
- As the number of possible rules dramatically increases with a number of states $\sigma$ and the size of the neighborhood $N$ ($\sigma^{|N|}$), it is usually non-trivial to find correct transition rules describing the system being modeled.
Examples 1D CAs

- $2^8 = 256$ 1D CAs
- Try rules 30, 54, 60, 62, 90, 94, 105, 150, 222, 250
- [https://ccl.northwestern.edu/netlogo/](https://ccl.northwestern.edu/netlogo/)
Example of 2D CA - Game of life -

• The transition rule consists of the evaluation of four independent logical conditions for each updated cell separately:
  ▫ a living cell having less than two living neighbors dies (loneliness);
  ▫ a living cell having more than three living neighbors dies (overcrowding);
  ▫ a living cell having two or three living neighbors stay living (ideal living conditions);
  ▫ a dead cell having exactly three living neighbors becomes alive (offspring).
Game of Life

Still Life - stable

Periodic Life Forms/Oscillators

Glider Gun

Garden of Eden
No parent could possibly produce the pattern.

Whether a given initial Life pattern can grow indefinitely, or whether any pattern at all can?
New glider every 30 generations
Game of Life and prime numbers

- **Primer** is a pattern that was constructed by Dean Hickerson on November 1, 1991. that produces a stream of lightweight spaceships representing prime numbers.
- N is prime if and only if a lightweight spaceship escapes to the left of the pentadecathlon at the bottom-left corner of the pattern at generation $120N+100$.
- The gun works by firing lightweight spaceships westward, and destroying them via glider guns that emulate the Sieve of Eratosthenes.
- [https://www.youtube.com/watch?time_continue=45&v=68nEX5CEmZE](https://www.youtube.com/watch?time_continue=45&v=68nEX5CEmZE)
Game of Life - logic gates

• Game of Life could be used to make functioning logic gates and perform operations on bit streams.
• Game of Life has a property of universality, meaning that it can compute anything that can be computed - Turing Complete
Segregation - example

• Exist two types of agents in a neighborhood, red and green.
• But each agent wants to make sure that it lives near some of “its own.”
• Each agent wants to live near at least some same type agents
Spatial isolation

- We have a two-dimensional lattice. In each cell there is an individual with probability $p$ or the cell is empty (probability $1-p$). Individuals have one of two different types. State of the cell is 0 if the cell is empty or has the state 1 or 2 depending on the type (sex, race, social class) for any individual in it. Local rule of evolution has three steps:
  - Each individual counts how many individuals have the same type in its environment (specifically, eight neighbors)
  - If this number is greater than or equal to 4 individuals are not move. If this is not the case, individuals are randomly moves to a neighboring cell
  - If two or more individuals decide to move it to the same cell, none of the individual is not moving.
Spatial isolation

• It starts from an arbitrary configuration. Below is shown the formation of spatial isolation
• Dissatisfied individuals are moving, causing the dissatisfaction of other individuals who are trying to move on. After many iterations of movement become illegal, some individuals remain trapped and unhappy in the vicinity of individuals of different types.
Levialdi
On shrinking binary picture patterns
Thank you for attention!